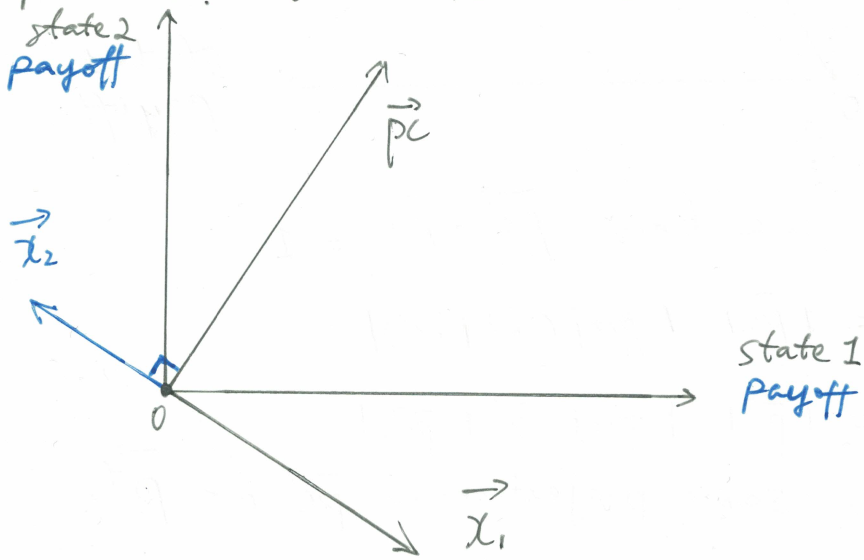
Figure 3.2

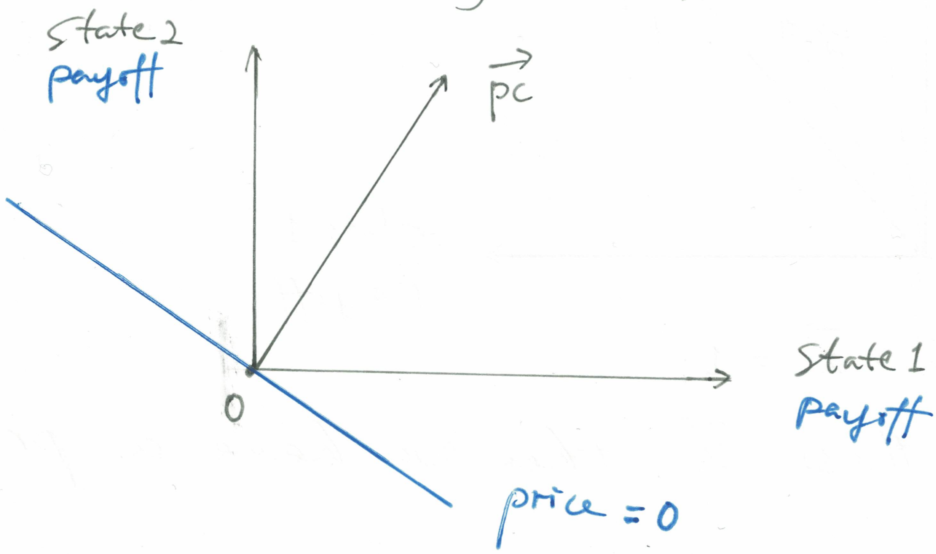
1. By definition $p(x) = \vec{pc} \cdot \vec{x}$

2. We consider the payoff space $\underline{X}$ of only two states (forming a plane). and $\vec{x} \in \underline{X}$



① Suppose we have a $\vec{pc}$ as above

② $\vec{pc} \cdot \vec{x_1} = \vec{pc} \cdot \vec{x_2} = 0$. If we collect all the payoffs like $\vec{x_1}$ & $\vec{x_2}$. then we find a price $=0$ payoffs plane.

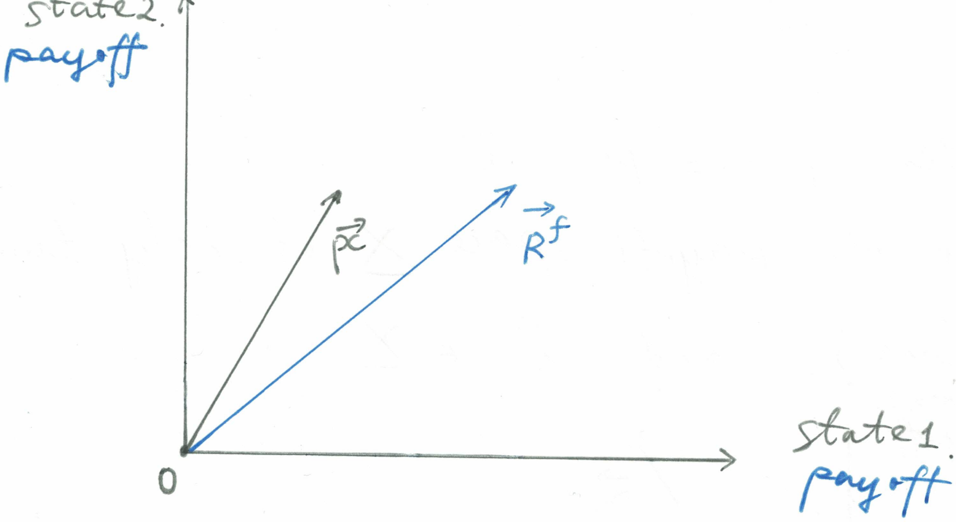


☆

3. Notice that $p(x) = \vec{pc} \cdot \vec{x} = |\vec{pc}| \cdot |\vec{x}| \cdot \cos(\theta) = |\vec{pc}| \cdot |proj(\vec{x}, \vec{pc})|$

It means if we have two vectors $\vec{x_1}$ and $\vec{x_2}$, and their projection on $\vec{pc}$ are the same, then $p(x_1) = \vec{pc} \cdot \vec{x_1} = \vec{pc} \cdot \vec{x_2} = p(x_2)$

Let's consider the price $=1$ payoffs plane.



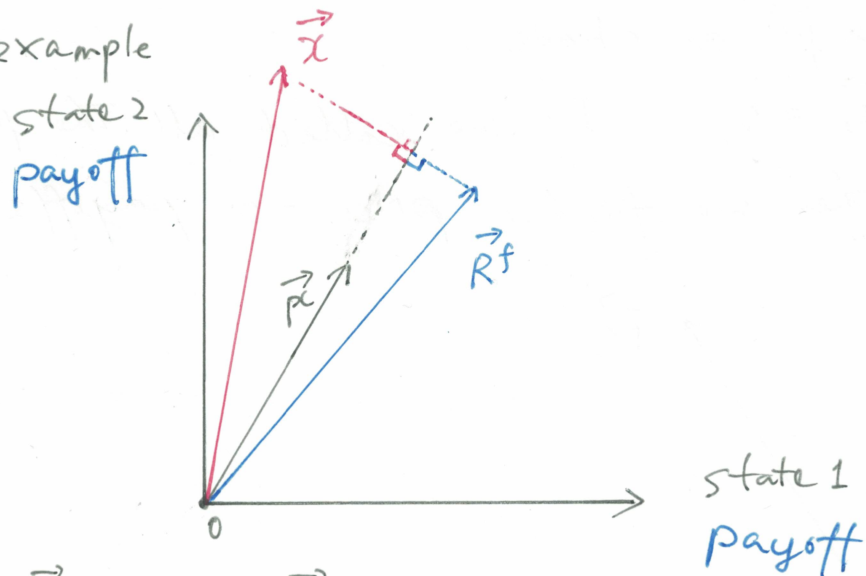
① By definition, we know $\vec{pc} \cdot \vec{R^f} = 1$

② $1 = \vec{pc} \cdot \vec{R^f} = |\vec{pc}| \cdot |proj(\vec{R^f} | \vec{pc})|$

$= |\vec{pc}| \cdot |proj(\vec{x} | \vec{pc})|$

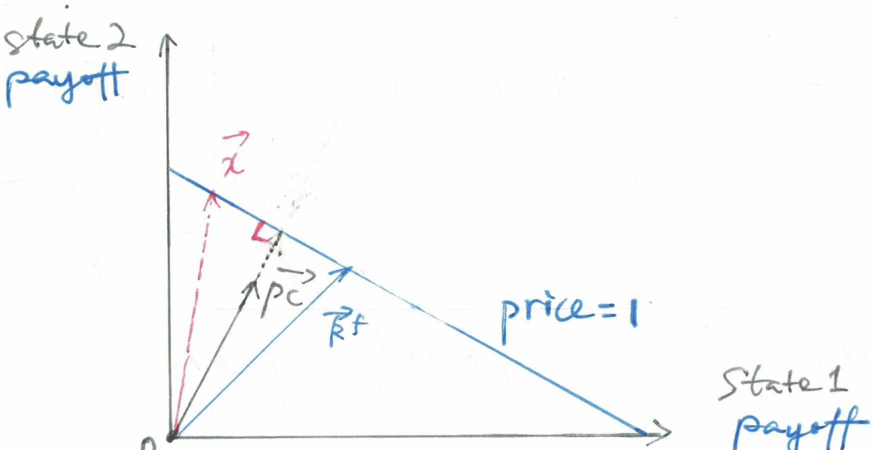
where $\vec{x}$ has the same projection on $\vec{pc}$ as $\vec{R^f}$.

③ for example



then $\vec{pc} \cdot \vec{R^f} = \vec{pc} \cdot \vec{x} = 1$.

We collect all these $\vec{x}$, then we have a price = 1 payoff plane.



4. Since all the price payoff planes are $\perp \overrightarrow{pc}$, they must be parallel. By the same construction, we can have any price planes.

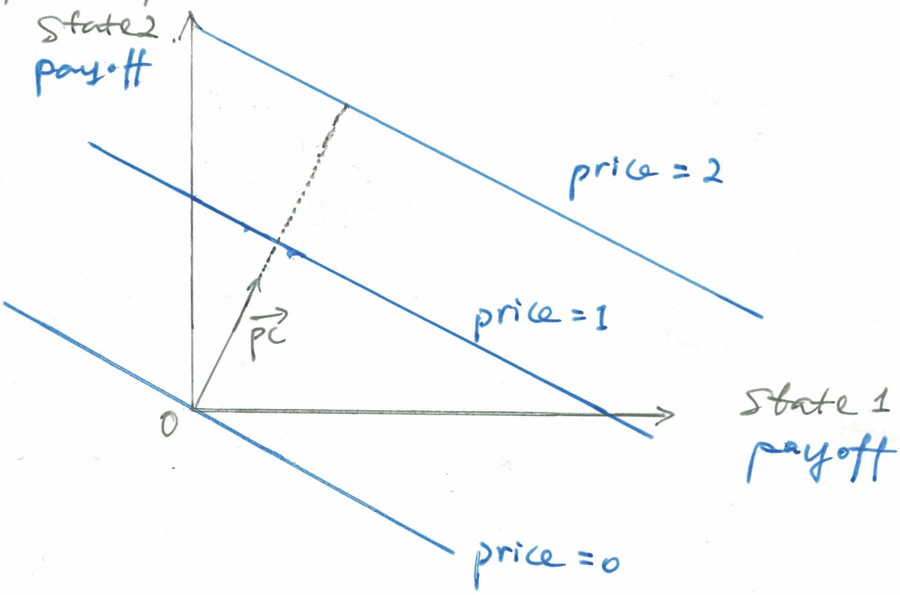
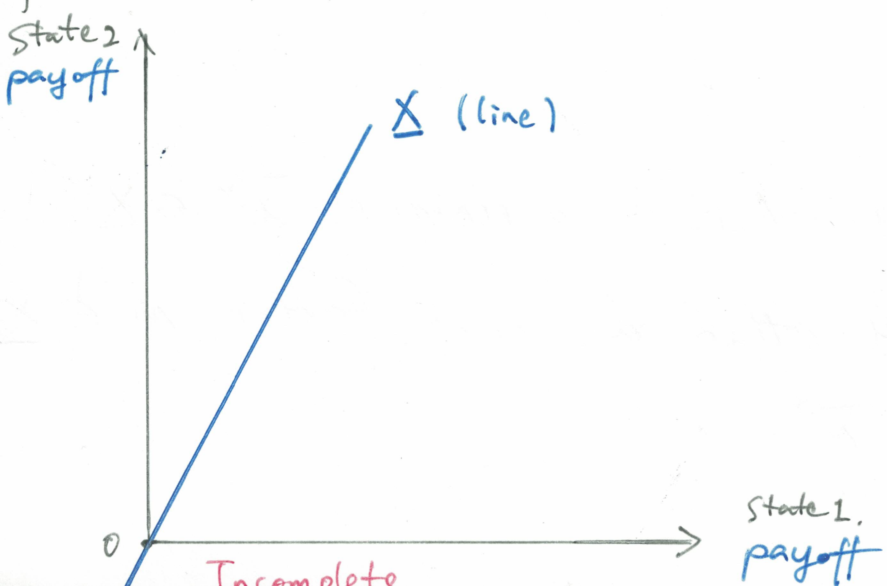


Figure 4.1

1. In a <u>complete</u> market, if we have K states, then $\underline{X} = \mathbb{R}^k$.
That is, if state space is a plane, then $\underline{X}$ is a plane.
if state space is a line, $\underline{X}$ is a line.

2. In a <u>incomplete</u> market, $\underline{X}$ must be subspace of $\mathbb{R}^k$.
That is, if state space is a plane, then $\underline{X}$ must be a line.
if state space is $\mathbb{R}^3$, then $\underline{X}$ can be a plane or a line.



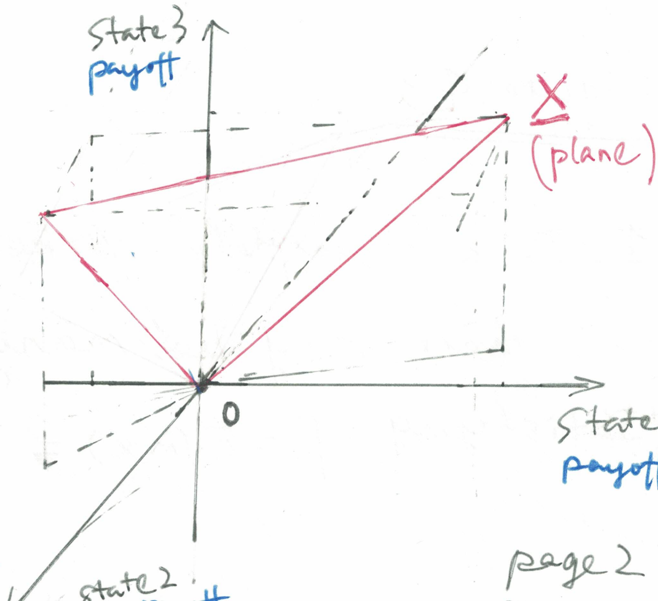
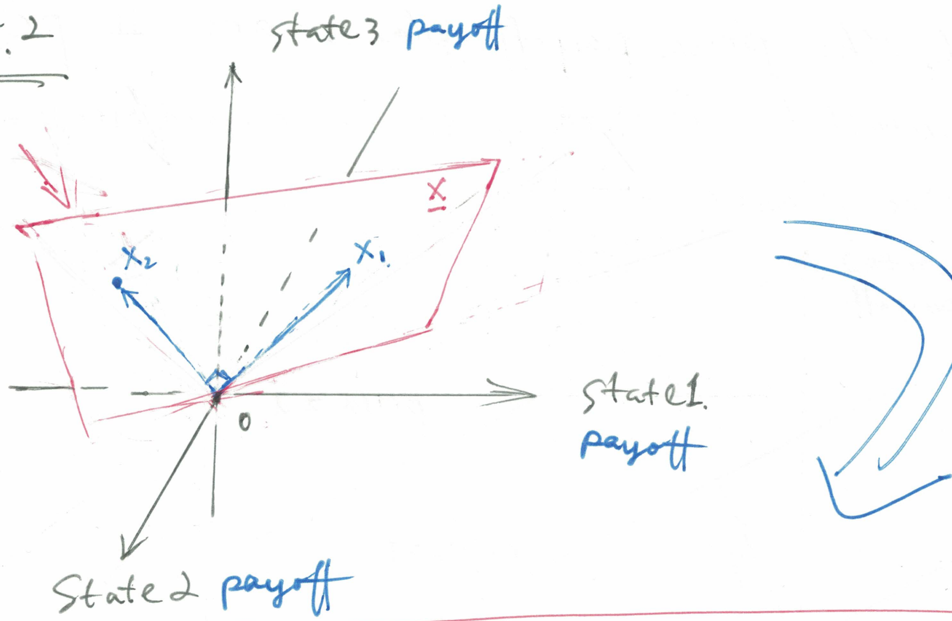
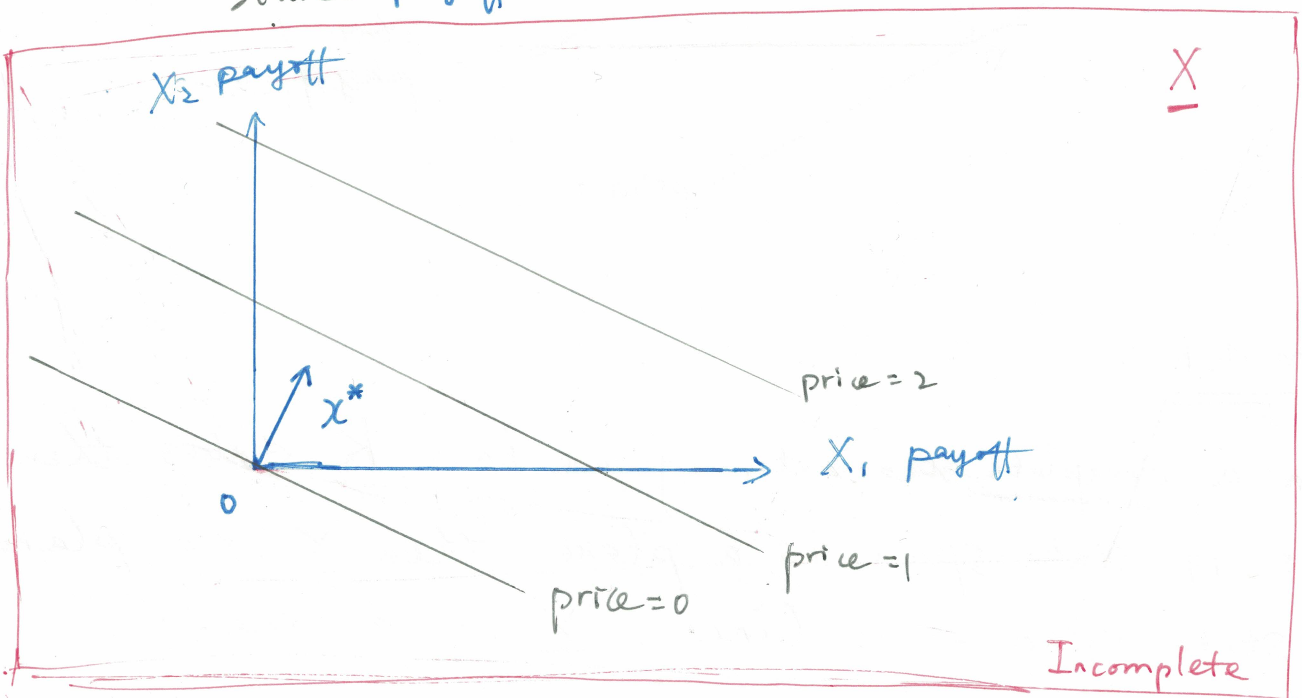


Figure 4.2



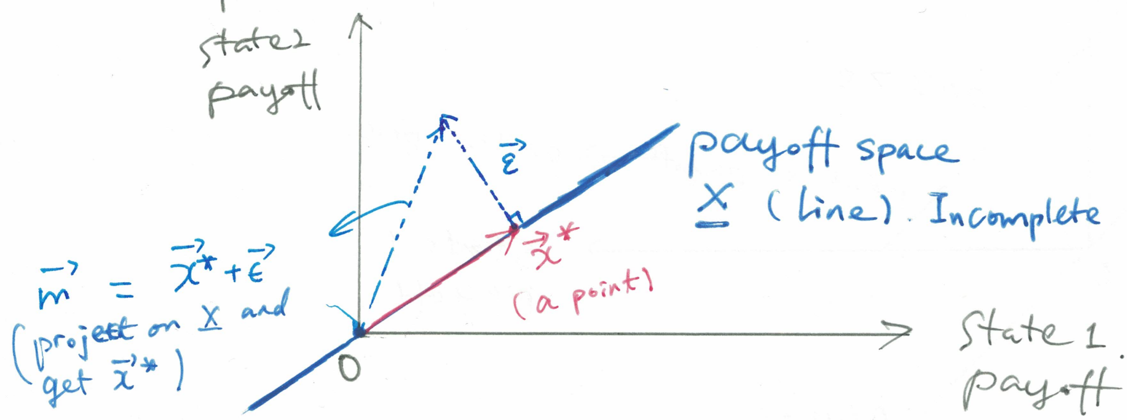


1. For any $\boxed{\text{LINEAR}}$ price, we can find a unique $\vec{x}^* \in \underline{X}$ and $\vec{x}^* \parallel \vec{p}(x)$, s.t. $p(x) = E(x^* x) = \vec{x}^* \cdot \vec{x}$

Figure 4.3

1. In a incomplete market, there is a unique $\vec{x}^* \in \underline{X}$, but there may be many other discount factor $m \notin \underline{X}$ satisfying $p = E(mx) = \vec{m} \cdot \vec{x}$

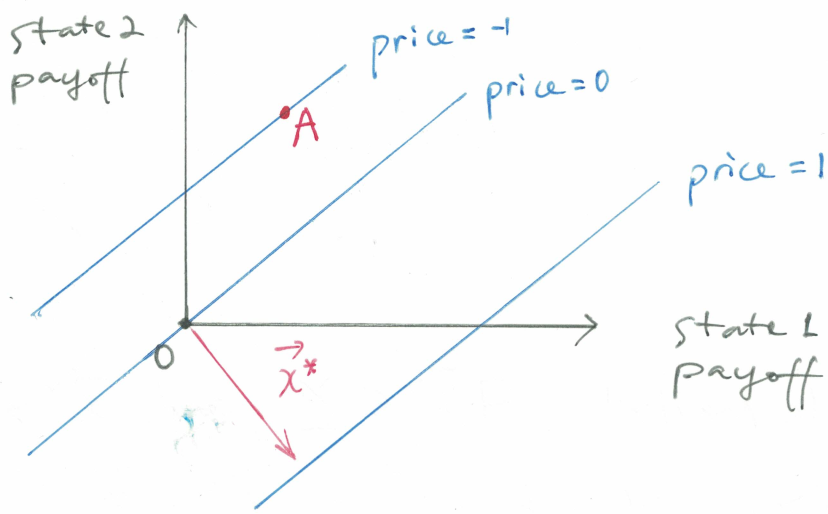
2. We consider a two states world, but the market is incomplete, then $\underline{X}$ is a line.



Then $\vec{m} \notin \underline{X}$, but $\vec{m} \cdot \vec{x} = \vec{x}^* \cdot \vec{x}$, $\forall \vec{x} \in \underline{X}$

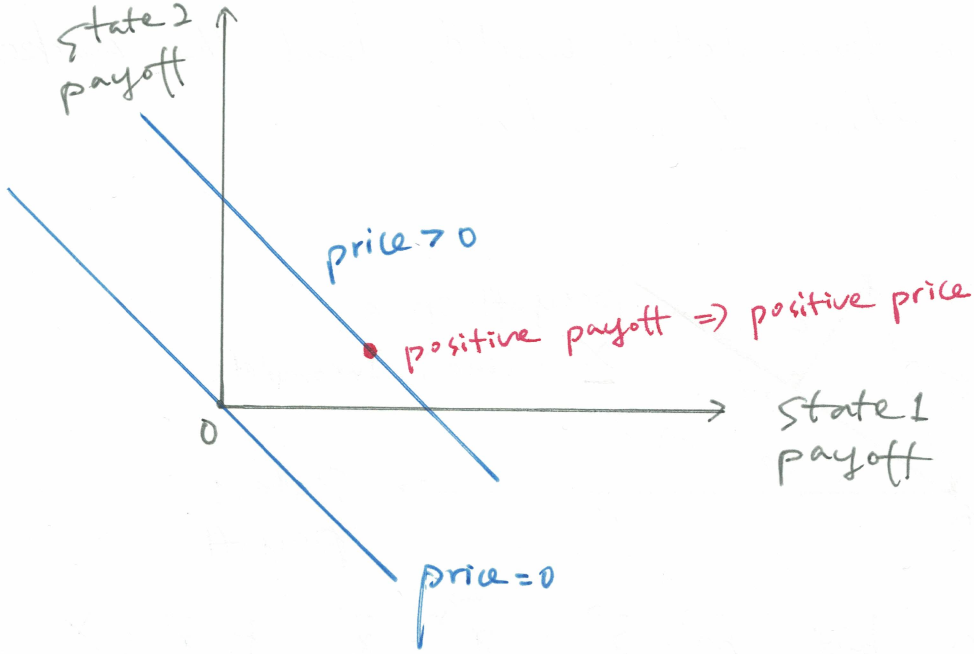
Figure 4.4

1. No arbitrage = positive payoff has positive prices
2. LOOP = linear price planes.
3. We consider a two states world

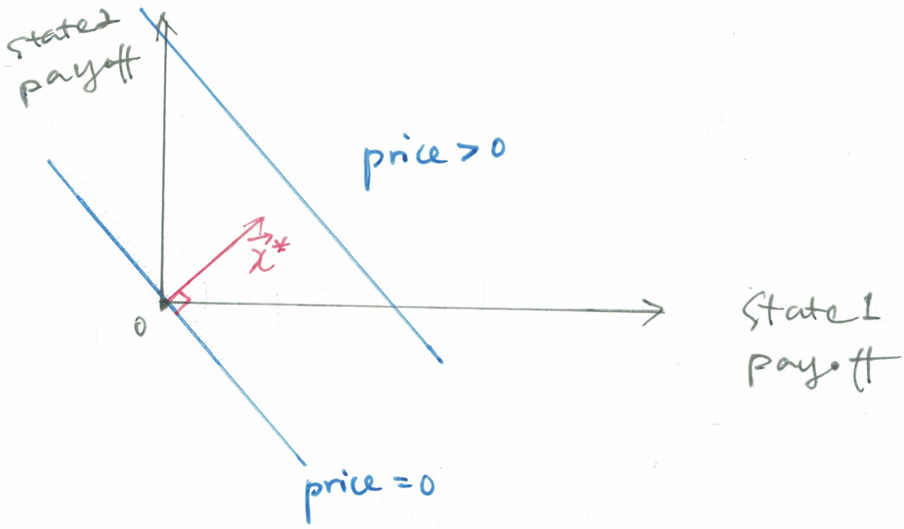


Then we cannot have this situation, because at A, we have positive payoffs without negative prices.

We MUST have



How $\vec{x}^*$ should be like?

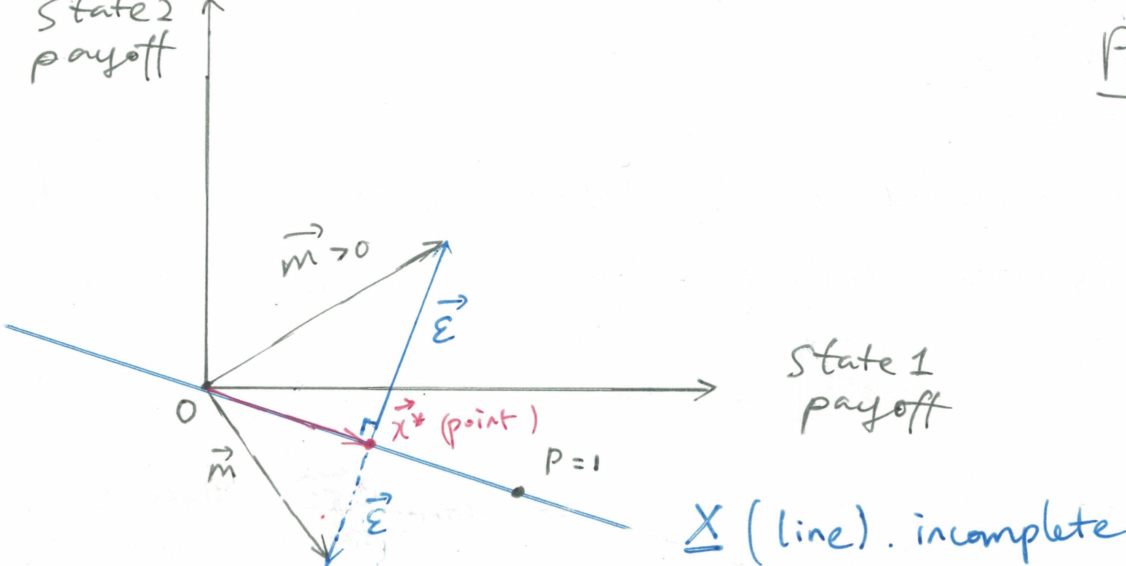


$\Rightarrow \vec{x}^* > 0$.

Figure 4.5

1. In a incomplete market, $\exists \vec{x}^* \in \underline{X}$ and $\vec{x}^* > 0$, but $\exists \vec{m} \notin \underline{X}$ and $\vec{m} > 0$ s.t. $\vec{m} \cdot \vec{x} = \vec{x}^* \cdot \vec{x}$

2. We consider a two states world (plane), then $\underline{X}$ is a line.



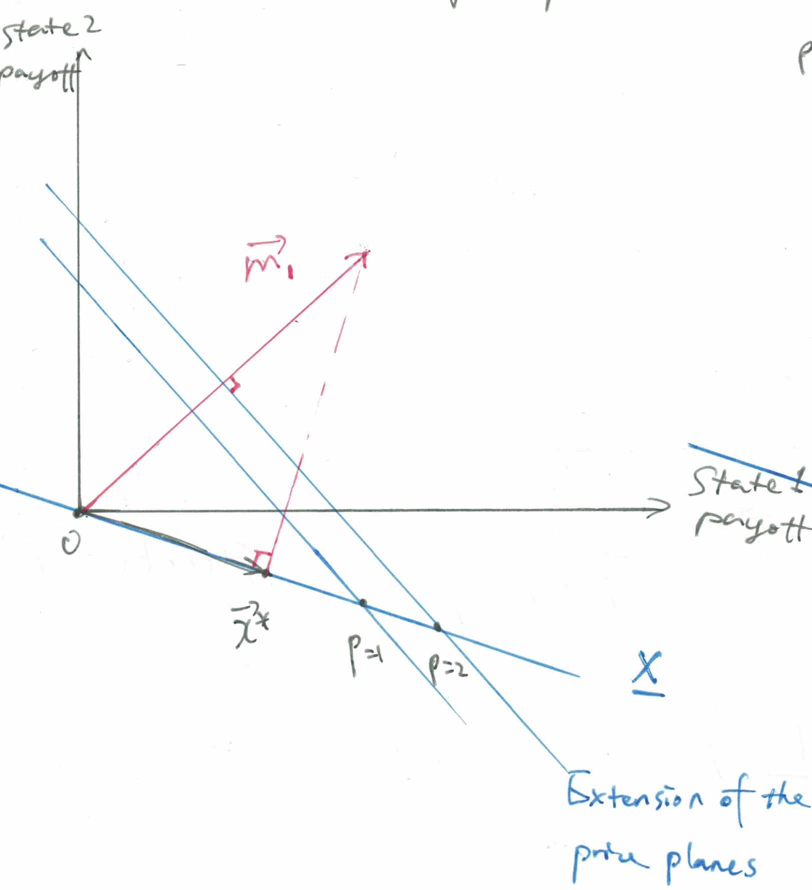
① Even though x^* is NOT positive, it does NOT contradict to no arbitrage if $p>0$ is on the right side of origin.

② It satisfies LOOP.

③ Then $\exists\, \vec{m} = \vec{x}^* + \vec{\varepsilon} \notin \underline{X}$ and $\vec{m} > 0$.

And $\vec{m}$ need not be positive.

3. If we extend the price planes, we can have different $\vec{m}$ create NO arbitrage opportunities.



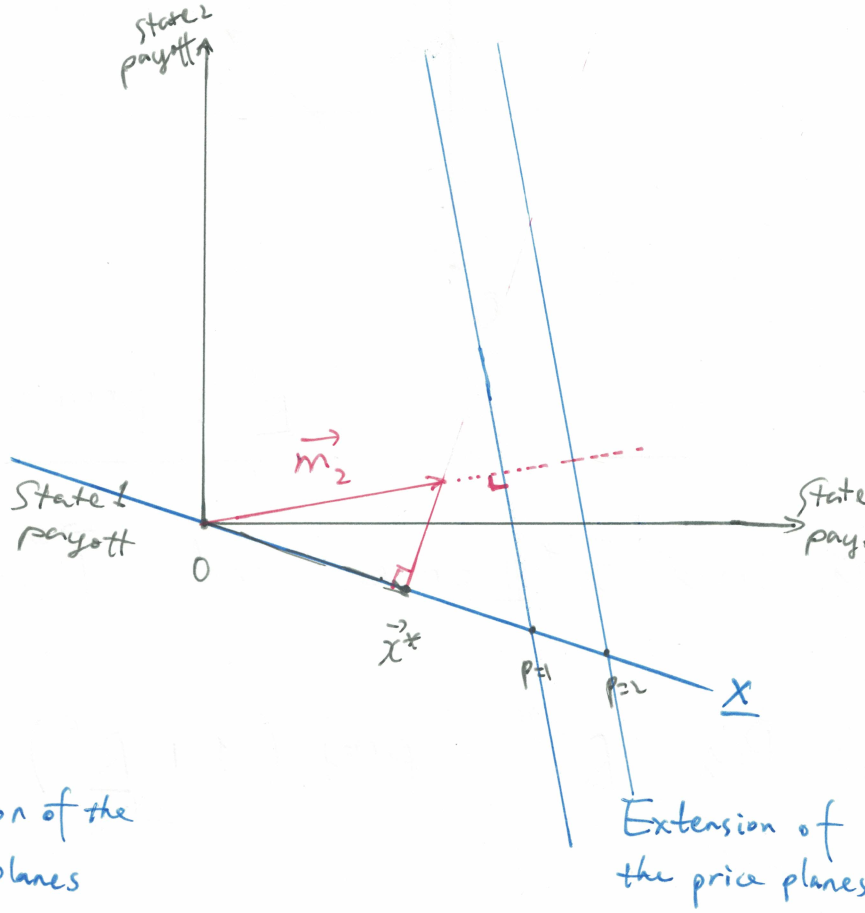
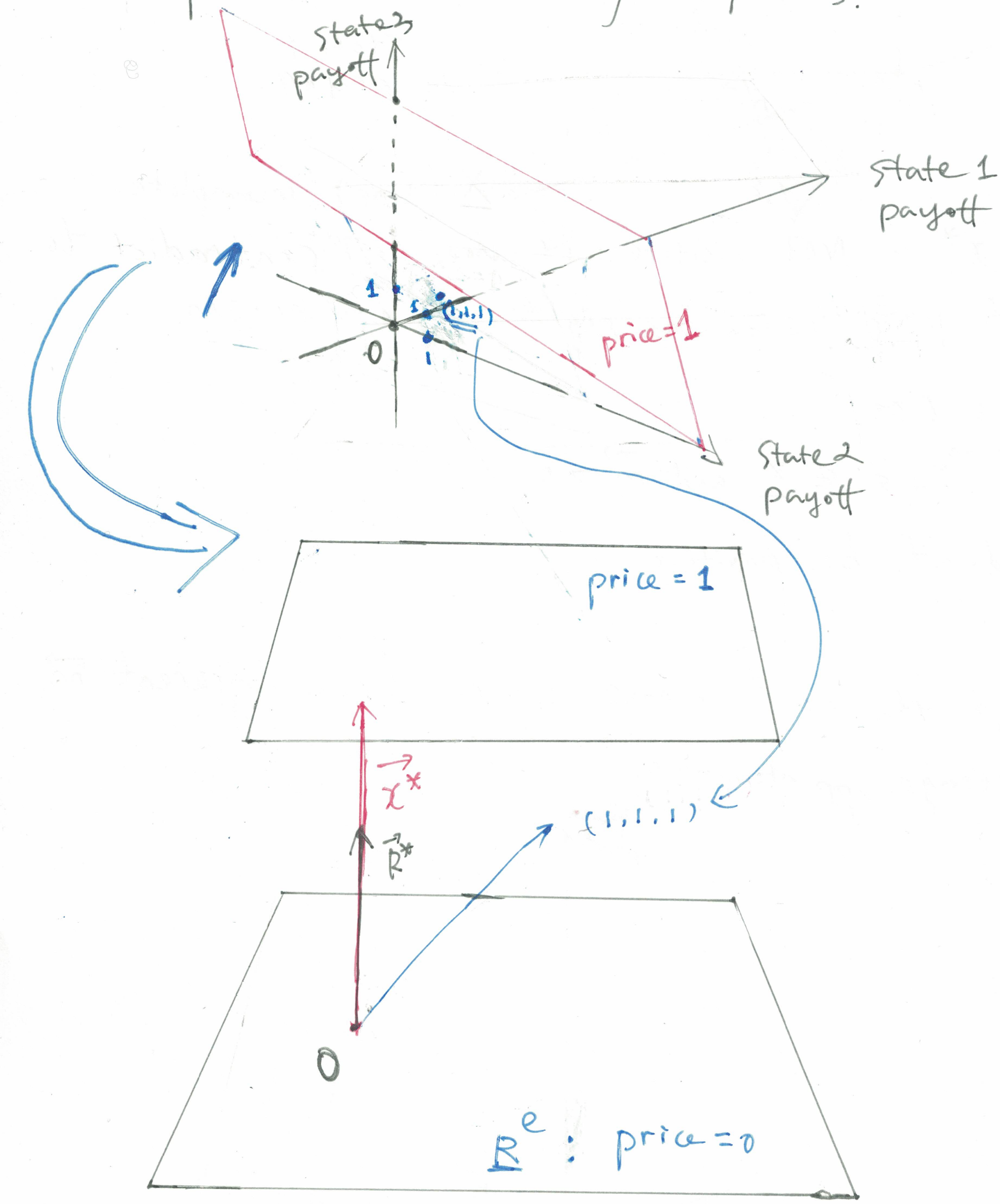


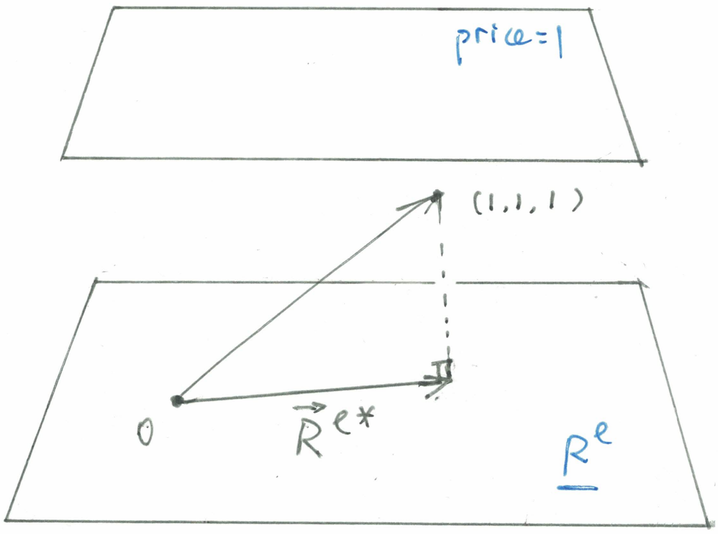
Figure 5.2

1. We consider a k states world, where $k \geq 3$.
The isoprice levels become flat planes.



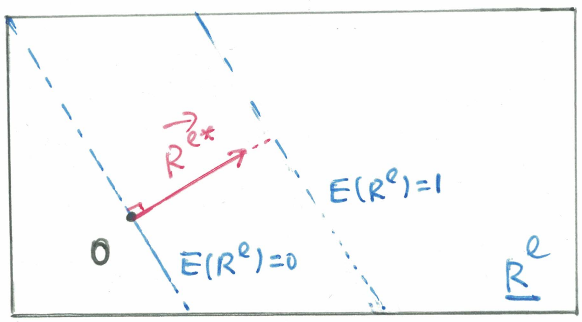
① Since $\vec{x}^* \perp$ price, and $\vec{R}^* = \dfrac{\vec{x}^*}{E(x^{*2})}$ just streches $\vec{x}^*$

2. By $R^{e*} = \text{proj}(1 \mid \underline{R}^e)$, we have.



3. Furthermore, by $E(R^e) = E(R^{e*}R^e) = \vec{R}^{e*} \cdot \vec{R}^e$

(As figure 3.2)



4. For any return R^i, it is clear that

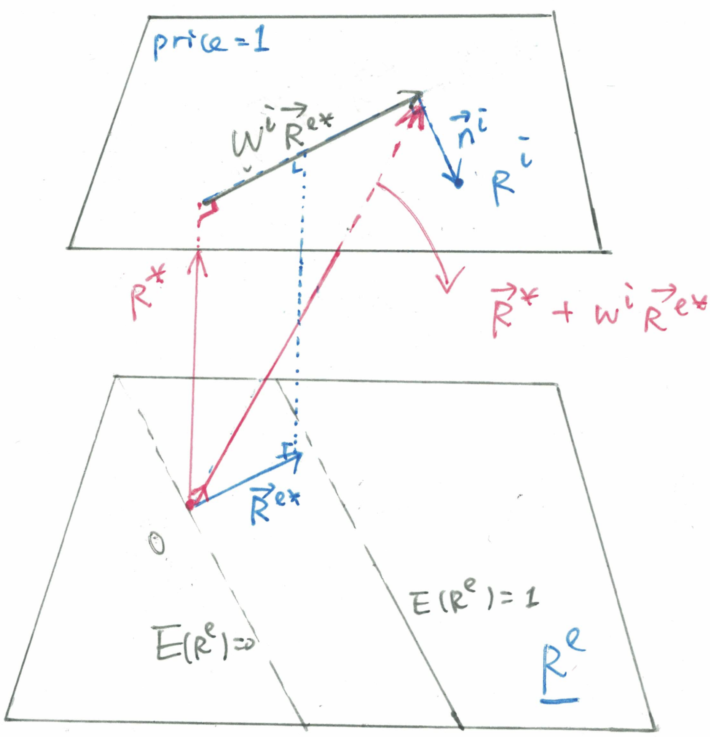


Figure 5.5 is basically the same as figure 5.2